Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ Total No. of Pages : 02

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B.Tech. (AI & DS / AI & ML / Block Chain / CE / CSE / CS / DS / CSD / EE / EEE / ETE / FT / IT / ME / Robotics & Artificial Intelligence / Internet of Things and Cyber Security including Block Chain Technology) (Sem.–2)

ENGINEERING MATHEMATICS-II

Subject Code : BTAM201/23

M.Code : 93811

Date of Examination : 09-06-2024

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Write briefly :

(a) Reduce in echelon form : $A = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 6 & 1 \\ 2 & 3 & 6 \end{pmatrix}$

(b) State Cayley-Hamilton theorem.

(c) Determine the value of k for which the homogeneous system of equations : $x - ky + z = 0;\ kx + 3y - kz = 0;\ 3x + y - z = 0$ has only trivial solution.

(d) Determine the eigen values of the matrix $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$.

(e) $T : \mathbb{R} \to \mathbb{R}^2$ defined by $T(x) = (2x, 3x)$. Check whether T is a Linear Transformation or not.

(f) Check the exactness of the differential equation : $e^x(\cos y\, dx - \sin y\, dy) = 0$.

(g) Solve the following differential equation $\sin x \frac{dy}{dx} + y \cos x = \cos x \sin^2 x$

(h) Check whether the Differential equation $y'' + yx^3 = 0$ is Linear and Non-linear?

(i) Solve the partial differential equation $(4D^3 - 3DD'^2 + D'^3)\, z = 0$.

(j) Eliminate the arbitrary constants a and b from $z = ax + by + a^2b^2$, to obtain the partial differential equation governing it.

SECTION-B

2. Solve the system of linear equations : $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, using inverse of a matrix.

3. Use Gauss-Jordon method to find the inverse of the matrix $A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$.

4. For the linear transformation $T : ¡^2 \to ¡^3$, defined by, $T(x, y) = (x + y, x - y, y)$, find a basis and dimension of (*i*) its range space and (*ii*) its null space. Hence verify rank-nullity theorem.

5. Let T be a linear operator on $¡^2$ defined by $T(x, y) = (4x - 2y, 2x + y)$.

(a) Find the matrix T relative to basis $B = \{(1, 1), (-1, 0)\}$.

(b) Also verify that $[T : B][v : B] = [T(v) : B]$ for any vector $v \in ¡^2$.

SECTION-C

6. Solve the differential equation $(D^2 + D + 1)\, y = \sin x$ using method of undetermined coefficients.

7. Solve $y' + 4xy + xy^3 = 0$.

8. Find the general solution of the partial differential equation $xy^2 p + y^3 q = (zxy^2 - 4x^3)$.

9. Solve $p = (qy + z)^2$ using Charpit's method.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

IKGPTU Exam June 2024

[ME, IOT, CSE, IT, AI-ML, ECE, AI-DS]

BTAM-201-23

Engineering Mathematics - II (M-II)

Solution of Ques Paper / June 2024

SECTION-A

(a) Reduce in echelon form: $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 6 & 1 \\ 2 & 3 & 6 \end{bmatrix}$

Soln:-

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 6 & 1 \\ 2 & 3 & 6 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 3 & -1 \\ 0 & -3 & 2 \end{bmatrix}$$

$R_2 \to R_2 + R_3$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & 1 \\ 0 & -3 & 2 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2/-3$

$$\sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}$$

) State Cayley-Hamilton Theorem

Ans:- Cayley Hamilton Theorem states that every square matrix satisfies its characteristic equation.

Example:- Let us suppose a matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

corresponding identity matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix}$$

The characteristic eq^n can be found out by using $|A-\lambda I| = 0$

$$|A-\lambda I| = (1-\lambda)(-1-\lambda) - 4 = 0$$

$$-1-\lambda+\lambda+\lambda^2-4 = 0$$

$$\lambda^2 - 5 = 0 \quad —①$$

Acc. to CHT, the eq^n ① becomes

$$A^2 - 5I = 0 \quad —②$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$5I = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Put value of A^2 and $5I$ in ②

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, verified that a square matrix always satisfy its characteristic equation.

(c) Determine the value of K for which the homogenous system of equation: $x - ky + z = 0$; $kx + 3y - kz = 0$; $3x + y - z = 0$ has only trivial solution.

Sol:- we can write the given eqⁿ in the form ~~AX=+~~

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$$1(-3+k) + k(-k+3k) + 1(k-9) \neq 0$$

$$k - 3 + 2k^2 + k - 9 \neq 0$$

$$2k^2 + 2k - 12 \neq 0$$

$$k^2 + k - 6 \neq 0$$

$$(k+3)(k-2) \neq 0$$

$$k \neq 2, -3$$

(d) Determine the eigen values of matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

Sol:- The given matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ is of order 2×2

The corresponding identity matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix}$$

The characteristic eqⁿ can be found out by using $|A - \lambda I| = 0$

$$|A - \lambda I| = (1-\lambda)(2-\lambda) - 12 = 0$$

$$2 - \lambda - 2\lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\lambda^2 - 5\lambda + 2\lambda - 10 = 0$$

$$\lambda(\lambda - 5) + 2(\lambda - 5) = 0$$

$$(\lambda + 2)(\lambda - 5) = 0$$

$\lambda = -2, 5$

$\therefore$ Eigen values are $-2, 5$.

) $T: R \to R^2$ defined by $T(x) = (2x, 3x)$. check whether T is linear transformation or not.

Solⁿ:- $T: \mathbb{R} \to \mathbb{R}^2$ is defined by $T(x) = (2x, 3x)$

for a transformation to be a L.T. It must satisfy the property.

$$T(\alpha v + \beta w) = \alpha T(v) + \beta T(w) \ \forall \alpha, \beta \in \mathbb{R}$$
$$v, w \in \mathbb{R}$$

Let $v = x$, $w = y \in \mathbb{R}$

$\Rightarrow$ $\alpha x + \beta y \in \mathbb{R}$ ($\mathbb{R}$ is VS)

$$T(\alpha x + \beta y) = [2(\alpha x + \beta y), 3(\alpha x + \beta y)]$$
$$= \alpha(2x, 3x) + \beta(2y, 3y)$$
$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$$

Hence T is Linear Transformation

) check the exactness of the differential eqⁿ:

$$e^x(\cos y\, dx - \sin y\, dy) = 0$$

Solⁿ:- For the equation to be exact it must satisfy the condition that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Eqⁿ:- $e^x(\cos y\, dx - \sin y\, dy) = 0$

can be written as

$$e^x \cos y\, dx - e^x \sin y\, dy = 0$$

here; $M = e^x \cos y$ and $N = -e^x \sin y$

$\frac{\partial M}{\partial y} = e^x(-\sin y)$ $\frac{\partial N}{\partial x} = \sin y(-e^x)$

$\frac{\partial M}{\partial y} = -e^x \sin y$ $\frac{\partial N}{\partial x} = -e^x \sin y$

here, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Therefore, it is exact differential equ

) solve the following differential equation

$\sin x \frac{dy}{dx} + y \cos x = \cos x \sin^2 x$

ans:- Divide the above equation by $\sin x$;

$\frac{dy}{dx} + y \cot x = \cos x \sin x$ — ①

Equate ① is of the form

$\frac{dy}{dx} + Py = Q$

where $P = \cot x$ and $Q = \sin x \cos x$

$\int P dx = \int \cot x \, dx = \log \sin x$

$IF = e^{\int P dx} = e^{\log \sin x} = \sin x$

general solⁿ is

$y.(I.F) = \int Q.(I.F) \, dx + c$

$y \sin x = \int \sin x \cos x (\sin x) \, dx + c$

$y \sin x = \int \sin^2 x \cos x \, dx + c$ — ②

$I = \int \sin^2 x \cos x \, dx.$

Put $\sin x = t \Rightarrow \cos x \, dx = dt$

$\therefore \quad I = \int t^2 dt$

$I = \frac{t^3}{3}$

$I = \frac{\sin^3 x}{3}$

Put value of I in (2)

$y \sin x = \frac{\sin^3 x}{3} + c$

therefore; general sol^n is

$y \sin x = \frac{\sin^3 x}{3} + c$

i) check whether the differential equation $y'' + yx^3 = 0$ is linear or not non-linea

sol:- The given Equation is

$$y'' + yx^3 = 0$$

The highest order is 2 i's $\frac{d^2y}{dx^2}$

and it's degree is 1

$\therefore$ The given Equation is linear

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i) solve the partial differential equation

$4D^3 - 3DD'^2 + D'^3 = 0$

sol^n:- Given differential eq^n is $4D^3 - 3DD'^2 + D'$

Put $D = \lambda$ and $D' = 1$

$4\lambda^3 - 3\lambda + 1 = 0$

Put $\lambda = -1$

-1	4	0	-3	1
	↓	-4	4	-1
	4	-4	1	0

$4\lambda^2 - 4\lambda + 1 = 0$

$4\lambda^2 - 2\lambda - 2\lambda + 1 = 0$

$2\lambda(2\lambda - 1) - 1(2\lambda - 1) = 0$

$(2\lambda - 1)(2\lambda - 1) = 0$

$\lambda = \frac{1}{2}, \frac{1}{2}$

$\therefore \lambda = -1, \frac{1}{2}, \frac{1}{2}$

CF $\Rightarrow$ $z = \phi_1(y - x) + x\phi_2(y + \frac{x}{2}) + \phi_3(y \cdot$

This is the required sol^n of given differential eq^n.

7) Eliminate the arbitrary constants a and b from $z = ax + by + a^2b^2$, to obtain the partial differential equation governing it.

sol^n:- $z = ax + by + a^2b^2$ — ①

Differentiate given eq ① wrt x

$$\frac{\partial z}{\partial x} = a$$

$p = a$ — ②

Diff. given eq^n ① wrt y

$$\frac{\partial z}{\partial y} = b$$

$q = b$ — ③

from ② and ③ put value of a, b in ①

$$\boxed{z = px + qy + p^2q^2}$$

ans:- 2 :- Solve the system of linear eq^n

$x + 2y - z = 3$

$3x - y + 2z = 1$

$2x - 2y + 3z = 2$ using inverse of Mx

Sol^n:-

$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

we known that

$AX = B$

$X = A^{-1}B$

As finding A^{-1}

$A^{-1} = \frac{adj\ A}{|A|}$

$|A| = 1[(-1)(3) - 2(-2)] - 2[3(3) - 2(2)] + (-1)[3(-2) + 2(-1)]$

$|A| = 1[-3+4] - 2[9-4] - 1[-6+2]$

$|A| = 1(1) - 2(5) - 1(-4)$

$|A| = 1 - 10 + 4$

$|A| = -10 + 5$

$|A| = -5$

Now,

finding Minors :-

$M_{11} = -3 + 4 = 1$

$M_{12} = 9 - 4 = 5$

$M_{13} = -6 + 2 = -4$

$M_{21} = 6 - 2 = 4$

$M_{22} = 3 + 2 = 5$

$M_{23} = -2 - 4 = -6$

$M_{31} = 4 - 1 = 3$

$M_{32} = 2 + 3 = 5$

$M_{33} = -1 - 6 = -7$

and cofactors:-

$A_{11} = 1$

$A_{12} = -5$

$A_{13} = 4$

$A_{21} = -4$

$A_{22} = 5$

$A_{23} = 6$

$A_{31} = 3$

$A_{32} = -5$

$A_{31} = -7$

$A = \begin{bmatrix} 1 & -5 & 4 \\ -4 & 5 & 6 \\ 3 & -5 & -7 \end{bmatrix}$

$adj\ A = \begin{bmatrix} 1 & -4 & 3 \\ -5 & 5 & -5 \\ 4 & 6 & -7 \end{bmatrix}$

Now,

$$A^{-1} = \frac{1}{-5}\begin{bmatrix} 1 & -4 & 3 \\ -5 & 5 & -5 \\ 4 & 6 & -7 \end{bmatrix} \Rightarrow \begin{bmatrix} -1/5 & 4/5 & -3/5 \\ 1 & -1 & 1 \\ -4/5 & -6/5 & -7/5 \end{bmatrix}$$

Now

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/5 & 4/5 & -3/5 \\ 1 & -1 & 1 \\ +4/5 & +6/5 & +7/5 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} + \frac{4}{5} - \frac{6}{5} \\ 3 - 1 + 2 \\ +\frac{12}{5} + \frac{6}{5} + \frac{14}{5} \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4-9}{5} \\ 4 \\ \frac{14+18}{5} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -4/5 \end{bmatrix}$$

two Mx are equal
1. order is same
2. element are same

so, $x = -1$

$y = 4$

$z = -4/5$

ans :- 3 use Gauss-Jordon Method find inverse of Mx

$$A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$$

Soln:-

$$A = AI$$

$$\begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$A^{-1} = \begin{bmatrix} -9 & -3 & 4 & 3 \\ 3 & -1 & -2 & -1 \\ -8 & -5 & 2 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

ans :- 4 $T: R^2 \to R^3$

$T(x,y) = (x+y, x-y, y)$

(i) Basis nd Dim

(ii) Range (T)

(iii) Nullity (T)

(iv) verify RNT :-

sol^n :- Given Transformation

$T: R^2 \to R^3$

$T(x,y) = (x+y, x-y, y)$

take usual basis of R^2

$B = \{(1,0), (0,1)\}$

now these Basis are span over R^2 and L.I

put Basis in $T(x,y)$

$T(1,0) = (1+0, 1-0, 0) = (1,1,0)$

$T(0,1) = (0+1, 0-1, 1) = (1,-1,1)$

now,

Range Basis are $(1,1,0), (1,-1,1)$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1/2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ $R_2 \to R_2/-2$

$\text{Range}(T) = \{(1,1,0), (0,1,-1/2)\}$

$\rho(T) = 2$

now nullity :-

$T(x,y) = (0,0)$

$(x+y, x-y, y) = (0,0,0)$

$x+y=0$

$x-y=0$ $\boxed{x=0}$

$\boxed{y=0}$

$$\begin{bmatrix}1&1&-1&-2\\0&2&1&3\\1&2&0&1\\-1&1&2&6\end{bmatrix}=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}A \Rightarrow \begin{bmatrix}1&1&-1&-2\\0&2&1&3\\0&3&2&7\\-1&1&2&6\end{bmatrix}=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&1\\0&0&0&0\end{bmatrix}A$$

$R_3 \rightarrow R_3 + R_4$ $\qquad$ $R_4 \rightarrow R_4 + R_1$

$$\begin{bmatrix}1&1&-1&-2\\0&2&1&3\\0&3&2&7\\0&2&1&4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&0\end{bmatrix}A \Rightarrow \begin{bmatrix}1&1&-1&-2\\0&1&1&4\\0&3&2&7\\0&2&1&4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\-1&0&1&1\\0&0&1&1\\0&1&0&0\end{bmatrix}A$$

~~$R_2 \rightarrow R_3 - R_2$~~

$R_4 \rightarrow R_4 - R_2$

$$\begin{bmatrix}1&1&-1&-2\\0&2&1&3\\0&3&2&7\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&1\\-1&1&0&0\end{bmatrix}A \Rightarrow \begin{bmatrix}1&1&-1&-2\\0&1&1&4\\0&3&2&7\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\-1&0&1&1\\0&0&1&1\\-1&1&0&0\end{bmatrix}A$$

$R_2 \rightarrow R_3 - R_2$ $\qquad$ $R_3 \rightarrow R_3 - 3R_2$

$$\begin{bmatrix}1&1&-1&-2\\0&1&1&4\\0&0&-1&-5\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\-1&0&1&1\\3&0&-2&-2\\-1&1&0&0\end{bmatrix}A \Rightarrow \begin{bmatrix}1&1&-1&-2\\0&1&0&-1\\0&0&-1&-5\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\2&0&-1&-1\\3&0&-2&-2\\-1&1&0&0\end{bmatrix}A$$

$R_2 \rightarrow R_2 + R_3$ $\qquad$ $R_2 \rightarrow R_2 + R_4$

$$\begin{bmatrix}1&1&-1&-2\\0&1&0&0\\0&0&-1&-5\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\3&1&-1&-1\\3&0&-2&-2\\1&1&0&0\end{bmatrix}A = \begin{bmatrix}1&1&-1&-2\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\3&1&-1&-1\\8&5&-2&-2\\1&1&0&0\end{bmatrix}\cdot 1$$

$R_3 \rightarrow R_3 + 5R_4$ $\qquad$ $R_3 \rightarrow R_3/-1$

$$\begin{bmatrix}1&1&-1&-2\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0&0\\3&1&-1&-1\\-8&-5&2&2\\1&1&0&0\end{bmatrix}A = \begin{bmatrix}1&1&-1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}2&3&0&0\\3&1&-2&-1\\-8&-5&2&2\\1&1&0&0\end{bmatrix}A$$

$R_1 \rightarrow R_1 + 2R_4$ $\qquad$ $R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix}1&1&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}-6&-2&2&2\\3&1&-2&-1\\-8&-5&2&2\\1&1&0&0\end{bmatrix}A \Rightarrow \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}-9&-3&4&3\\3&1&-2&-1\\-8&-5&2&2\\1&1&0&0\end{bmatrix}A$$

$R_1 \rightarrow R_1 - R_2$

Nullity $(T) = 0$

now RNT

$\rho(T) + N(T) = \dim(V)$

$2 + 0 = 2$

$2 = 2$ Verify.

ans:- 5:. $T: R^2 \to R^2$

$T(x,y) = (4x - 2y, 2x + y)$

(i) MY T related Basis - $\{(1,1), (-1,0)\}$

(ii) verify $[T:B][V:B] = [T(V):B] \ \forall \ V \in R^3$

Soln:-

(i) $B = \{(1,1), (-1,0)\}$

form linear combination

$x_1 = (1,1)$

$x_2 = (-1,0)$

$(x,y) = \alpha_1(1,1) + \alpha_2(-1,0)$ — (#)

~~(x,y)~~ $\alpha_1 - \alpha_2 = x$ — (1)

$\boxed{\alpha_1 = y}$ —(2)

put (2) in (1)

$y - \alpha_2 = x$

$\boxed{y - x = \alpha_2}$ — (3)

put (3) nd (2) in (#)

$(x,y) = y(1,1) + (y-x)(-1,0)$ — ☆

now put Basis in T

$T(1,1) = (4-2, 2+1) = (2,3)$ =

$T(-1,0) = (-4, -2) = (-4,-2)$ =

~~now~~, now put $(2,3)$ nd $(-4,-2)$ in ☆

$(x,y) = 3(1,1) + 1(-1,0)$

$(x,y) = (-2)(1,1) + 2(-1,0)$

Now $M_x T = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$

$$T' = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$$

SECTION – C

6. $(D^2 + D + 1)y = \sin x$ — ① (by undetermined coefficient method)

$A.E \Rightarrow D^2 + D + 1 = 0$

$\Rightarrow D = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\therefore y_c = e^{-\frac{1}{2}x}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$

Let $y_p = A\cos x + B\sin x$

$\Rightarrow y' = -A\sin x + B\cos x$

$\Rightarrow y'' = -A\cos x - B\sin x$

$\therefore$ From ①, we have

$-A\cos x - B\sin x - A\sin x + B\cos x + A\cos x + B\sin x = \sin x$

$\Rightarrow -A\sin x + B\cos x = \sin x$

$\Rightarrow -A = 1 \; ; \; B = 0$

$\Rightarrow \boxed{A = -1}, \boxed{B = 0}$

$\therefore y_p = -\cos x$

Hence, complete solution is

$y = y_c + y_p$

$\Rightarrow \boxed{y = e^{-\frac{1}{2}x}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right] - \cos x}$

Sol:- 7. $y' + 4xy + xy^3 = 0$

$\Rightarrow \frac{dy}{dx} + 4xy = -xy^3$

which is of the form $\frac{dy}{dx} + P.y = Q.y^n$

$\frac{1}{y^3}\frac{dy}{dx} + 4x \cdot \frac{1}{y^2} = -x$ — ①

Put $\frac{1}{y^2} = t \Rightarrow y^{-2} = t$

$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dt}{dx}$

∴ From ①, we have

$-\frac{1}{2}\frac{dt}{dx} + 4xt = -x$

$\Rightarrow \frac{dt}{dx} - (8x)t = 2x$

which is of the form $\frac{dt}{dx} + P.t = Q$

where, $P = -8x$, $Q = 2x$

∴ $I.F = e^{\int P dx} = e^{\int -8x\, dx} = e^{-8x^2/2} = e^{-4x^2}$

$\Rightarrow t \cdot (I.F) = \int Q \cdot (I.F)\, dx + c$

$\Rightarrow \frac{1}{y^2} e^{-4x^2} = \int e^{-4x^2} \cdot 2x\, dx + c$

Put $-4x^2 = u$

$\Rightarrow -4 \cdot 2x\, dx = du$

$\Rightarrow 2x\, dx = \frac{-1}{4} du$

$$\therefore \frac{1}{y^2} e^{-4x^2} = \frac{-1}{4}\int e^u du + c$$

$$\Rightarrow \frac{1}{y^2} e^{-4x^2} = -\frac{1}{4} e^u + c$$

$$\Rightarrow \boxed{\frac{1}{y^2} e^{-4x^2} = -\frac{1}{4} e^{-4x^2} + c} \text{ Ans.}$$

Sol:-8 $xy^2p + y^3q = (2xy^2 - 4x^3)$

Comparing with Lagrange's equation

$$Pp + Qq = R$$

Here, $P = xy^2$; $Q = y^3$; $R = (2xy^2 - 4x^3)$

$\therefore$ Lagrange's A.E. are :-

$$\frac{dx}{xy^2} = \frac{dy}{y^3} = \frac{dz}{2xy^2 - 4x^3} \quad -(1)$$

Taking first two members of A.E, we have

$$\frac{dx}{xy^2} = \frac{dy}{y^3}$$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{y}$$

Integrating both sides,

$$\ln|x| = \ln|y| + \ln|c|$$

$$\Rightarrow \ln|x| - \ln|y| = \ln|c|$$

$$\Rightarrow \boxed{c_1 = \frac{x}{y}} \quad -(2)$$

Taking last two members of A.E, we have

$$\frac{dy}{y^3} = \frac{dz}{zxy^2 - 4x^3}$$

Put $x = c_1 y$ from eq. (2)

$$\Rightarrow \frac{dy}{y^3} = \frac{dz}{z \cdot c_1 y \cdot y^2 - 4c_1^3 y^3}$$

$$\Rightarrow dy = \frac{dz}{zc_1 - 4c_1^3}$$

$$\Rightarrow c_1 dy = \frac{dz}{z - 4c_1^2}$$

Integrating both sides;

$$c_1 y = \ln|z - 4c_1^2| + c_2$$

$$\Rightarrow \boxed{x - \ln\left|z - 4\frac{x^2}{y^2}\right| = c_2}$$

$\therefore$ General solution is given by

$$\boxed{f\left(\frac{x}{y},\ x - \ln\left|z - 4\frac{x^2}{y^2}\right|\right) = 0}$$

Solve $p = (qy + z)^2$ — (1) using Charpit's method.

The auxiliary equations are given by

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}}$$

$$= \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} \quad -(2)$$

Let $f = (qy+z)^2 - p = 0$

Now, $\frac{\partial f}{\partial x} = 0$ $\qquad \frac{\partial f}{\partial p} = -1$

$\frac{\partial f}{\partial y} = 2q(qy+z)$ $\qquad \frac{\partial f}{\partial q} = 2y(qy+z)$

$\frac{\partial f}{\partial z} = 2(qy+z)$

$\therefore$ We have

$$\frac{dp}{2p(qy+z)} = \frac{dq}{4q(qy+z)} = \frac{dz}{(-p)(-1) - q\cdot 2(qy+z)y}$$

$$= \frac{dx}{-(-1)} = \frac{dy}{-2y(qy+z)}$$

Taking first and last member,

$$\frac{dp}{p} + \frac{dy}{y} = 0$$

$\therefore$ $\log p + \log y = \log a$

$$\Rightarrow \boxed{p = \frac{a}{y}}$$

Now, From ① $\Rightarrow \frac{a}{y} = (qy+z)^2$

$$\Rightarrow \sqrt{\frac{a}{y}} = qy+z$$

$$\Rightarrow \boxed{q = \frac{\sqrt{a}}{y^{3/2}} - \frac{z}{y}}$$

Now, $dz = p\,dx + q\,dy$

$$\therefore \; dz = \frac{a}{y}\,dx + \left(\frac{\sqrt{a}}{y^{3/2}} - \frac{z}{y}\right) dy$$

$$\Rightarrow \; y\,dz + z\,dy = a\,dx + \frac{\sqrt{a}}{\sqrt{y}}\,dy$$

$$\Rightarrow \; \boxed{yz = ax + 2\sqrt{ay} + b} \quad \text{Ans.}$$